1. We can find the space diagonal by finding a face diagonal, and doing Pythagorean's with a face diagonal and an edge. A face diagonal would be $\sqrt{3^2+3^2}=3\sqrt{2}$. Thus, a space diagonal would be $\sqrt{(3\sqrt{2})^2+3^2}=\sqrt{18+9}=\boxed{3\sqrt{3}}$

2. First, convert the side length to the correct unit: 30 centimeters = 0.3 meters. Then, cube the side length. $(0.3)^3=\boxed{0.027 \text{ or } \frac{27}{1000}}$.

3. The average sum of a single dice roll can be calculated by finding the sum of all possible outcomes, and dividing it by the amount of different outcomes. This results in $\dfrac{1+2+3+4+5+6}{6}=\dfrac{21}{6}=\dfrac{7}{2}$. In order to find the average sum of three fairly rolled dice would be $3\cdot\dfrac{7}{2}=\boxed{\dfrac{21}{2} \text{ or } 10.5}$.

4. We need to find the sum of all possible outcomes, and divide it by the amount of different outcomes. Because of the distributive property, the sum of all possible outcomes is $(1+2+3+4+5+6)(1+2+3+4+5+6)=21^2=441$. The total amount of outcomes is $6\cdot 6=36$. $\dfrac{441}{36}=\boxed{\dfrac{49}{4} \text{ or } 12.25}$

5. Note that 4096 is 2 raised to the 12^{th} power. This means that 4096 expressed in binary is 1 followed by 12 zeroes. If we subtract 1, it will become $\boxed{12}$ ones.

6. The polynomial equates to $f(x)=x^2+3x+2$. Using power rule, we can convert this into $f'(x)=2x+3$. Evaluating for the slope at $x=6$ leads us to $2(6)+3=\boxed{15}$.

7. The sum of the first n cubes is equivalent to the sum of the first n natural numbers, squared. This cancels out, leaving only $\sum_{i=1}^{20} i$ in the denominator, which is equal to $20(21)/2=210$ for a final answer of $\boxed{\dfrac{1}{210}}$.

8. Rose curve equations are of the form $r=a\sin(n\theta)$ or $r=a\cos(n\theta)$. If n is even, the curve has $2n$ petals, and if n is odd, the curve has n petals. This means that rose curves can have any odd number of petals, and any even number that is twice another even number. However, the only even numbers that are twice another even number are divisible by 4. Thus, rose curves cannot have n petals if n is even and not divisible by 4, or more simply, $n\equiv 2 \mod 4$. Thus, from 2 to 21, the only values of n that do not work are 2, 6, 10, 14, and 18. There are 15 values that work out of 20, for a probability of $\boxed{\dfrac{3}{4} \text{ or } 0.75}$.

9. We can solve this with the following identity: $(a-b)(a+b)=a^2-b^2$. Thus, $(100-1)(100+1)=10000-1=\boxed{9999}$.

10. The sum of the eigenvalues of a matrix is always equal to the trace of the matrix. The trace of the matrix is $4+9+6+1=\boxed{20}$.

11. $3^7=\boxed{2187}$.

12. Due to the bracelet being able to rotate and flip congruently, we are able to produce 6 rotations and 2 flips at all times. The amount of ways to arrange 6 distinct objects is 6!, or 720. Thus, the answer is $\dfrac{720}{6\cdot 2}=\boxed{60}$.

13. We can use the same trick as in problem 9. $(95+1)(95-1)=9025-1=\boxed{9024}$.

14. Note that whole numbers include 0. The sum of the first n squares (starting at 1) is $\frac{(n)(n+1)(2n+1)}{6}$. Therefore, the answer is $\frac{(9)(10)(19)}{6} = \boxed{285}$.

15. The largest prime under 2021 is $\boxed{2017}$.

16. We begin by noticing the following pattern:

$$\frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \ldots + \frac{1}{575} = \frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \ldots + \frac{1}{23 \cdot 25}.$$

Take note that each pair of numbers differs by 2, so if we divide and multiply the equation by 2, we can write the numerator in terms of the denominator.

$$= \frac{1}{2}\Big(\frac{2}{1 \cdot 3} + \frac{2}{3 \cdot 5} + \frac{2}{5 \cdot 7} + \ldots + \frac{2}{23 \cdot 25}\Big) = \frac{1}{2}\Big(\frac{3-1}{1 \cdot 3} + \frac{5-3}{3 \cdot 5} + \frac{7-5}{5 \cdot 7} + \ldots + \frac{25-23}{23 \cdot 25}\Big)$$

Now, we can write the numbers as a telescoping sequence.

$$= \frac{1}{2}\Big(\frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \ldots + \frac{1}{23} - \frac{1}{25}\Big) = \frac{1}{2}\Big(\frac{1}{1} - \frac{1}{25}\Big) = \frac{1}{2} \cdot \frac{24}{25} = \boxed{\frac{12}{25} \text{ or } 0.48}$$

17. A dodecagon has 12 sides, so the number of diagonals in a convex dodecagon is $\frac{n(n-3)}{2} = \frac{12(9)}{2} = \boxed{54}$.

18. $\cos(\frac{\pi}{3}) = \boxed{0.5 \text{ or } \frac{1}{2}}$

19. 2020 was a leap year, so it had 366 days. The amount of days that would shift forward in terms of day of the week would be $366 \equiv 2 \pmod 7$. Therefore, the first day of 2021 was Friday (2 days after Wednesday). 2021 was not a leap year, so it had 365 days. The amount of days that would shift forward is $365 \equiv 1 \pmod 7$. One day after Friday is $\boxed{\text{Saturday}}$.

20. The product of eigenvalues is equivalent to the determinant of the matrix. Because the third row is a multiple of the first row, the determinant of the matrix is $\boxed{0}$.

21. The external angle of a regular n-gon is equivalent to $\frac{360}{n}$. We need to find the amount of values for n for which the external angle (and thus, the internal angle) is an integer. The prime factorization of 360 is $2^3 \cdot 3^2 \cdot 5^1$. This means that 360 has (3+1)(2+1)(1+1) = 24 factors. However, a polygon cannot have 1 or 2 sides, so the answer is $\boxed{22}$.

22. $(6)\binom{6}{2}(10) = \frac{(6)(6)(5)(10)}{2} = \boxed{900}$

23. The sum of the first 10 elements is $10^3 = 1000$. The sum of the first 9 elements is $9^3 = 729$. Therefore, the 10^{th} element is $1000 - 729 = \boxed{271}$

24. The sum of the first n odd natural numbers is n^2. Therefore, the given expression is equal to $73^2 - 27^2$. From the identity discussed in problem 9, we know this is $(73+27)(73-27) = (100)(46) = \boxed{4600}$.

25. The prime factorization of 2021 is $43 \cdot 47$. This means that 2021 has $\boxed{3}$ factors excluding itself: 1, 43, and 47.

26. This can be solved by imagining he sleeps 4 hours all 30 days. This means he sleeps 120 hours. The deficit is $208 - 120 = 88$ hours, meaning that he sleeps 12 hours on $\frac{88}{12-4} = \boxed{11}$ days.

27. The product of the LCM and GCF of two numbers is equivalent to the product of the two numbers. $(50+5)(50-5) = 50^2 - 5^2 = \boxed{2475}$.

28. The units digit of 482^{29} is equivalent to the units digit of 2^{29}. We notice that the unit digits of powers of two cycle in the following manner: 2, 4, 6, 8, 2, 4, 6, 8, ... Because 29 is equivalent to 1 mod 4, the units digit of 2^{29} is 2. The units digits of all powers of 5 is 5, and the units digits of all powers of 6 is 6. Therefore, the answer is $2 + 6 + 5 = 10 + \boxed{3}$.

29. The area of a rectangular prism of side lengths a, b, and c is $2(ab + bc + ac)$. We can find $ab + bc + ac$ via Vieta's, by finding the sum of the roots taken 2 at a time. The sum of the roots taken 2 at a time will be $\frac{496}{10}$. Multiplying this by 2 will give us $\boxed{\frac{496}{5} \text{ or } 99.2}$.

30. $-\frac{b}{2a}$ tells us the maximizing or minimizing value for x. In this case, $-\frac{b}{2a} = 2$. $2^2 - 4(2) + 20 = \boxed{16}$.

31. This is a typical stars and bars problem. In this case, we have 5 stars (problems) and 2 bars (3 - 1 people to separate them between). $\binom{5+2}{2} = \frac{7 * 6}{2} = \boxed{21}$.

32. $\frac{5! + 3!}{4! + 4!} = \frac{126}{48} = \boxed{\frac{21}{8} \text{ or } 2.625}$

33. Envision the 9 by 4 inch base of the suitcase. We can fit 3 shirts in a 9 by 4 by 1 inch space, and leave a 1 by 9 inch space aside. We do this for 5 layers, fitting 15 shirts. We are left with a 5 by 9 inch space. We will fit 3 shirts here, leaving a 2 by 9 inch space aside, which cannot be filled with anything other than USB drives. We will be able to fit $\boxed{18}$ USB drives.

34. $2^2 \cdot \pi \approx 4 \cdot 3.14 = 12.56 \approx \boxed{13}$

35. 2021 both began and ended on a Friday. This means that every day of the week other than Friday happened 52 times in 2021. Thus, our answer is $365 \cdot 2 - 52 \cdot 2 = 313 \cdot 2 = \boxed{626}$.

36. The volume of the room is $10^3 = 1000$. The volume of one eighth of a sphere with radius 10 is $\frac{1}{8} \cdot \frac{4}{3} \cdot 1000\pi = \frac{500\pi}{3}$. The volume remaining for Ananya is $\boxed{1000 - \frac{500\pi}{3}}$.

37. The prime factorization of 2016 is $2^5 \cdot 3^2 \cdot 7$. The answer is $2 \cdot 5 + 3 \cdot 2 + 7 \cdot 1 = 10 + 6 + 7 = \boxed{23}$.

38. The shape created is a square with a side length of $10\sqrt{2}$. $(10\sqrt{2})^2 = \boxed{200}$.

39. $20 + 21 = \boxed{41}$.

40. The probability of the Queen of Hearts being the fifth card from the top is the same as the probability of it being anywhere in the deck. Therefore, the answer is $\boxed{\frac{1}{52}}$